The Fundamental Electrical Charge on a Series of Oil Droplets

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Abstract

The electric charge on a series of oil droplets, calculated from measuring the motion of each individual droplet with or without an electric field present, demonstrates that charge is quantized as an integer factor of a fundamental charge. This charge is presumed to be that of an electron, with the value $Q = (1.62 \pm 0.06) \times 10^{-19}$ C.

I. INTRODUCTION

In previous methods of deducing the fundamental electric charge, the value was computed from the average behavior of grouped charges in electrical and magnetic fields. The earliest method executed by H.A. Wilson measured the rise and fall velocity of a charged water vapor cloud with and without an electric field, respectively, and calculated the average charge to be between 6.67×10^{-20} C and 1.33×10^{-19} C [1]. Following this, J.J. Thomson deflected cathode rays in the presence of a magnetic field. He discovered the deflection is equivalent to that on a negatively charged body moving along the ray's path by a magnetic force, and concluded that cathode rays are composed of negatively charged particles with the value 1.27×10^{-19} C [2]. I will refer to these particles as 'electrons', a name proposed by G. Johnson Stoney for the amount of electricity passing through a 1 Ampere current each second [3]. The search for the electron's charge continued again with electric fields. For example, F. Ehrenhaft measured 1.53×10^{-19} C from the average motion of colloid particles of phosphorous with no field and of metals with the field [4]. Similarly, the quantity 1.50×10^{-19} C was measured by L. de Broglie from the velocities of charged tobacco smoke particles in an electric field [5]. The experiment presented in this paper was undertaken to improve measurement accuracy by removing error from averaging the group motion of charge carriers. Here, an electric field is used to control droplet motion. The charge is computed from the average motion of individual droplets without or without an electric field present, unlike the previous methods.

II. APPARATUS

Oil droplets are sprayed from an atomizer, which electrically charges droplets via friction, into an oil vapor tower above an observing chamber. These droplets have either positive or negative sign due to complicated and not well understood processes occurring when the oil is atomized. Below the vapor tower, there is a chamber illuminated by a laser. It consists of an upper plate with small holes for the droplets to pass through, and a lower plate. Applying a DC voltage to the metallic plates produces a uniform electric field, which can be manually turned off and on, and the direction of the field lines, perpendicular to the plate surfaces, can be reversed. The chamber has plastic sides that separate the plates and maintain their parallelism, and glass separators at the front and back. A Charged Coupled Device (CCD) captures a focused live video through a monitor. In the video feed, the distance scale is calibrated by an array of 2 μm tungsten wires which have a center-to-center spacing of 0.3175 mm with negligibly small uncertainty. The wires are set by the pitch of the #0-80 screws around which they are wound and their image is reflected from the front glass plate of the chamber, which is in plane with the falling oil droplets. This reflection forms the virtual image of the wires as focused on the CCD camera, with negligible magnification uncertainty.

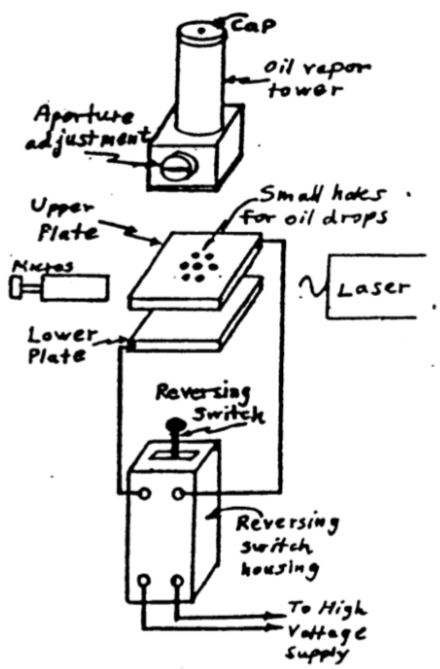

FIG. 1: Oil droplets travel down the oil vapor tower and fall through the small holes on the upper plate. The position of the reversing switch can be controlled to make the droplets between the plate, visible via laser, fall or rise.

III. METHODS

To deduce the electron charge, the charges on a series of oil droplets must be determined. Measurement of charge on a droplet relies on its falling speed in the absence of an electric field and its rising speed in the presence of an electric field. Let us consider the first scenario. Without an electric field, the net force on a falling droplet at speed ν is:

$$F_{\text{drag}} - F_{\text{buoyant}} = 6\pi\eta r\nu - \frac{4}{3}\pi r^3 \left(\rho_{\text{oil}} - \rho_{\text{air}}\right) g \tag{1}$$

In equilibrium, the net force is zero and the droplet has a terminal velocity ν_1. Setting the net force to zero, we obtain:

$$\frac{4}{3}\pi r^3 \left(\rho_{\text{oil}} - \rho_{\text{air}}\right) g = 6\pi\eta r\nu_1 \tag{2}$$

From Equation 2, we derive the oil droplet radius to be:

$$r = \sqrt{\frac{9\eta\nu_1}{2\rho g}}, \text{ where } \rho = \left(\rho_{\text{oil}} - \rho_{\text{air}}\right) \approx \rho_{\text{oil}} \tag{3}$$

Therefore, only the falling velocity ν_1 is required to determine the droplet's radius. Let us now consider the effect of an electric field given by:

$$E = \frac{V}{D} \tag{4}$$

Where V is the voltage difference between the upper and lower plate separated by distance D. The net force on the rising droplet at speed ν is:

$$F_{\text{electric}} - F_{\text{drag}} - F_{\text{buoyant}} = q\frac{V}{D} - 6\pi\eta r\nu - \frac{4}{3}\pi r^3\rho g \tag{5}$$

Note that $F_{\text{drag}} < 0$ allows the droplet to move upward. In equilibrium, the net force is zero and the drop has a terminal velocity ν_2. Setting the net force to zero, we obtain:

$$q\frac{V}{D} = \frac{4}{3}\pi r^3\rho g + 6\pi\eta r\nu_2 = 6\pi\eta r(\nu_1 + \nu_2) \tag{6}$$

Now, substituting Equation 3 for r, the droplet's charge is given by:

$$q = \frac{D}{V}6\pi\eta r(\nu_1 + \nu_2) = \frac{18\pi D}{V}\sqrt{\frac{\eta^3\nu_1}{2\rho g}}(\nu_1 + \nu 2) \tag{7}$$

Note that a correction on η is required because r is comparable to the mean free path. In this case, the droplets are small enough such that F_{drag} on the droplet is less than it would be if calculated based on the viscosity η_0 of a large droplet. The viscosity correction is given by:

$$\eta = \frac{\eta_0}{1 + \frac{b}{pr}} \tag{8}$$

Where the mean free path in the air $b = 6.17 \times 10^{-5}\text{m} \cdot \text{Torr}$, the temperature-dependent viscosity $\eta = 1.827 \times 10^{-6}\sqrt{\frac{T}{291}\frac{\text{kg}}{\text{m}\cdot\text{s}}}$, and p is the atmospheric pressure. After this correction on Equation 7, we can define two coefficients:

$$C_1 = 18\pi D\sqrt{\frac{\eta^3}{2\rho g}} \tag{9}$$

$$C_2 = (1 + b/pr)^{-3/2} \tag{10}$$

Where C_1 is the same for all droplets and characterized by the experimental setup and C_2 is unique for each droplet. The independent variables which remain are ν_1 and ν_2. Thus, the droplet charge defined as:

$$q = C_1\sqrt{\nu_1}\,(\nu_1 + \nu_2)\,\frac{C_2}{V} \tag{11}$$

Thus, the falling and rising velocities ν_1 and ν_2 are required to determine the charge on an oil droplet. These terminal velocities are determined experimentally as the distance between two wires divided by the average time to fall or rise between them.

The experimental procedure is devised to record the rise and fall times on a series of oil droplets. We first verify the camera setup by passing a pin through the central hole of the top plate. The pin and horizontal tungsten wires must be in the focal plane of the camera because this is the plane in which most of the oil droplets will be falling, and the wires are used to scale the droplet displacement. Once this is confirmed, the atmospheric pressure and room temperature are recorded. This measurement will be taken again at the end of the experiment. Next, oil droplets of density $\rho = 869.42\frac{\text{g}}{\text{L}}$ are sprayed a few times into a paper towel until a good spray is obtained. Then, they are sprayed once into the vapor tower. It is important not to inject too much oil into the tower because it can cause the formation of large droplets, which are not ideal. Instead, we are looking for smaller and slower droplets which will most likely have a smaller charge. If charge is quantized, this will make charge quantization more obvious by having smaller integer factors of a base charge. Following this, the voltage supply is turned on and set to approximately 500 V. Turning the voltage off, the falling droplets are observed in zero electric field until a very slowly falling drop is found, one that takes approximately 10-20 seconds to fall the screen length. The voltage supply is turned on to verify the drop is charged, and the polarity of the electric field between the plates which are separated by D = 4.46×10^{-3}m is chosen such that the droplet will rise upward when the field is applied. We trap the very slow falling charged oil droplet in the field of view by applying a voltage before it reaches the bottom of the screen, and adjusting the voltage so the rise time is conveniently long. A voltage difference of 478 ± 1 V is found to be optimal. We measure a series of fifteen droplets, with the fall and rise times measured ten times for each.

If charge is quantized, the charges on a series of droplets should each be an integer multiple n of the electron charge. Each droplet charge will be assigned a factor n based on how they sort into histogram bins. From this, the average base charge for each n is calculated, and the values can be compared for consistency with one another. For the number of droplets N in a series of charges q_i where $i = 1, 2, ..., N$, we define:

$$x_i = \frac{q_i}{Q} \tag{12}$$

This represents the ratio of the charge to an unknown base charge Q. If charge is quantized, the ratio will be an integer. Therefore, to determine the optimal Q for this data, we use the integer n_i closest to x_i and compute Chi-Square defined as:

$$\chi^2 = \sum_i^N (x_i - n_i)^2 \tag{13}$$

If the droplet charges are exact multiples of Q, χ^2 would be zero. Thus, we compute χ^2 for a list of 10,000 values of Q in a range enclosing the experimental averages. If there is a χ^2 with a unique and minimum that is approximately zero, its corresponding Q is the base value for quantized charge, presumably the electron charge.

IV. RESULTS

The calculated charge on each droplet is assigned an integer factor n determined by the range of charge values for each bin in Fig.1. The integer n represents the quantity of base charge, presumably the electron charge, contained by the droplet (Table 1). The average was then computed for each group of charges sorted by n (Table 2). Droplets with charge factors $n = 2$ and $n = 3$ have larger uncertainty than for $n = 1$ because there are less data points for these droplets and because they travel faster due to their larger mass, which reduces time measurement accuracy. In Table 2, the range of averages provide an estimate the electron charge: between 1.4×10^{-19} C and 1.6×10^{-19} C.

The χ^2 fit, given by Equation 13, depends on the difference between a base charge integer n and a ratio of the charge to an unknown base charge Q (Eq. 12). Figure 3 shows there is a minimum $\chi^2 = 0.05508$ at $Q = 1.62 \times 10^{-19}$ C, which is the best approximation of a fundamental charge for the data set. The total uncertainty on Q is computed from the variation in its value when χ^2 is increased by a factor of two, relative to its minimum, which produces the fundamental charge result $Q = (1.62 \pm 0.06) \times 10^{-19}$ C.

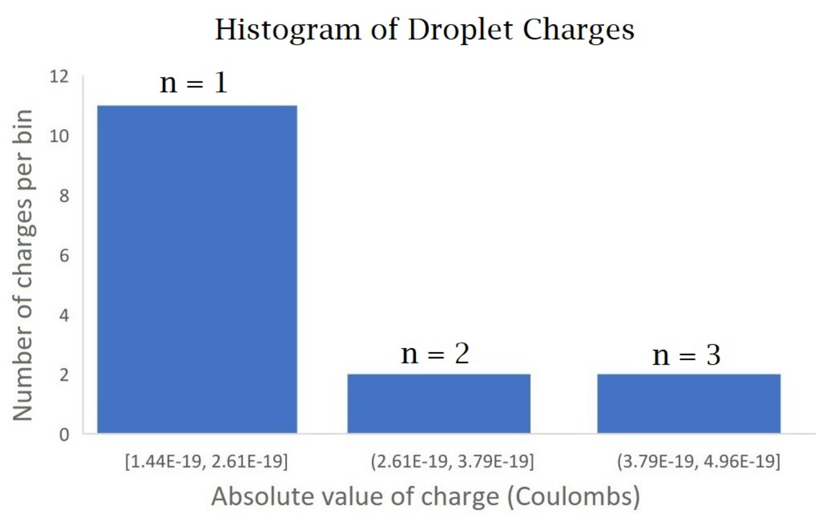

FIG. 2: Histogram of the droplet charges. Each bin is assigned an integer $n = 1, 2, 3$ to represent the quantity of base charge contained by the droplet.

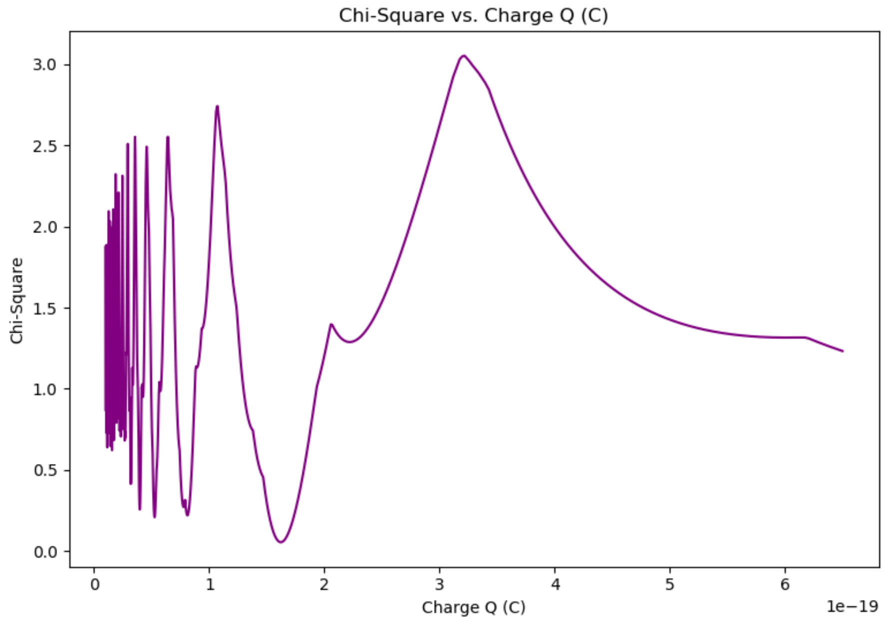

FIG. 3: χ^2 versus possible droplet base charges in the range $(0.1 \times 10^{-19}, 6.5 \times 10^{-19})$ C.

V. SUMMARY

The electron charge is determined to be $Q = (1.62 \pm 0.06) \times 10^{-19}$ C, which is at the upper limit of the predicted range from the Table 2 values. It is higher than the charge values from the experiments by J.J. Thompson [2], F. Erenhaft [4], and L. de Broglie [5]. This suggests their experimental uncertainty, due to measuring the average motion of a system rather than on individual parts in the system, reduces the magnitude of their measured fundamental charge. However, in our experiment, the most significant source of uncertainty is the time measurement because the drops were difficult to observe. This would signify that the experimental time is measured to be longer than the actual time for the droplets to rise or fall. Regardless, the χ^2 plot (Fig.1) suggests that charge is quantized because χ^2 is minimized when the measured charges are close to their integer factor of Q. We infer, there exists a fundamental charge Q for which the series of charges can be divided by to produce almost an exact integer, which suggests the electron charge contained by the oil droplets is discrete.

VI. TABLES

Droplet	Fall Velocity ($10^{-5} \cdot \frac{\text{m}}{\text{s}}$)	Rise Velocity ($10^{-5} \cdot \frac{\text{m}}{\text{s}}$)	Charge ($10^{-19} \cdot$ C)	n
1	12.0 ± 0.4	4.11 ± 0.20	$+4.96 \pm 0.23$	3
2	5.74 ± 0.33	2.33 ± 0.22	-1.61 ± 0.12	1
3	4.99 ± 0.28	3.20 ± 0.25	-1.50 ± 0.10	1
4	2.97 ± 0.25	8.15 ± 0.41	$+1.48 \pm 0.11$	1
5	3.51 ± 0.18	6.62 ± 0.35	$+1.50 \pm 0.08$	1
6	4.74 ± 0.28	3.72 ± 0.12	-1.50 ± 0.10	1
7	2.45 ± 0.21	9.75 ± 0.41	$+1.44 \pm 0.10$	1
8	3.19 ± 0.27	8.01 ± 0.33	-1.56 ± 0.11	1
9	2.88 ± 0.68	8.25 ± 0.29	-1.45 ± 0.05	3
10	11.7 ± 0.5	3.64 ± 0.13	$+4.65 \pm 0.26$	1
11	3.64 ± 0.24	6.30 ± 0.27	$+1.50 \pm 0.10$	1
12	4.10 ± 0.21	5.05 ± 0.37	$+1.49 \pm 0.10$	1
13	3.81 ± 0.13	5.99 ± 0.28	-1.53 ± 0.07	1
14	5.21 ± 0.19	10.3 ± 0.5	-2.92 ± 0.13	2
15	3.16 ± 0.20	17.8 ± 0.9	$+2.90 \pm 0.17$	2

TABLE I: Droplet charges, calculated from rise and fall velocities, assigned an integer n to represent the quantity of base charge contained by the droplet.

Charge quantity	Average Charge ($10^{-19} \cdot$ C)
$n = 1$	1.51 ± 0.03
$n = 2$	1.46 ± 0.11
$n = 3$	1.60 ± 0.17

TABLE II: Average charge for each n.

[1] H.A. Wilson, Phil. Mag., 5, p. 429 (1903).

[2] J.J. Thomson, Cathode Rays, Philosophical Magazine, 44, 293 (1897).

[3] G. J. Stoney, On the Cause of Double Lines and of Equidistant Satellites in the Spectra of Gases, The Scientific Transactions of the Royal Dublin Society (2nd ser.) 4, 563–608 (1888–1892).

[4] F. Ehrenhaft, Phys. Zeit., Mai, (1909).

[5] L. de Broglie, Le Radium, Juillet, (1909).