Observed Quantized Conductance Between Gold Wires

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Abstract: The goal of our experiment was to observe quantized conductance by placing two gold wires in a circuit with an applied voltage. Data is collected as voltage traces versus time on an oscilloscope to measure the conductance response as integer multiples of $2e^2/h$. We observed the conductance integer multiples as step sizes represented by peaks on histograms of voltage data. Our measured step sizes compared with the theoretical values of step sizes gave a lower percent error for lower integer multiples of the conductance. Solving for the conductance of all voltage data, we found our measured quantum conductance, $G_0 = 7.72 * 10^{-5} \pm 0.08 * 10^{-5}$, to be within uncertainty to the theoretical value, $G = 7.75 * 10^{-5}$.

Background

Nanostructures demonstrate quantum mechanics in their electrical, mechanical, and optical properties. These properties, not found in larger devices, can be used as the bases for future technologies. A simple apparatus can be created to model nanostructure physics to explore its effects [2]. One important effect is the quantization of electrical conductance.

The effect of quantized conductance was first discovered by the Dutch group at the same time experimenters at Cambridge University were observing the phenomena. The groups were both studying point-contact spectroscopy with different motivations. Point-contact spectroscopy is important for the study of electrons scattering in a solid object [1]. As the two points touched, the contact suggested quantization of conductance as it demonstrated a step-like behavior when provided with a negative voltage. This discovery is similar to the quantum Hall effect. In this effect, an electron gas in the presence of a magnetic field also shows a stepping pattern at different multiples of e^2/h for a decreasing magnetic field. Experimenters attempted to answer the question as to why the conductance changes in integral multiples as two electron reservoirs are connected. They found that the point-contacts act as a waveguide for the electrons. When the contact is varied, the allowed number of modes that can move into the waveguide will jump in steps discontinuously [2]. For each allowed mode there is a conductance of integer multiples of $2e^2/h$, which can be derived.

Derivation

The current of a wire, I, of length L is determined by the number N and velocity v of electrons with charge e, as follows:

$$I = veN/L \quad \textbf{(1)}$$

The conductance of a wire, G, is the current divided by the supplied voltage:

$$G = I/V \quad \textbf{(2)}$$

Thus, the conductance can be re-written in terms of Eq.1,

$$G = veN/LV \quad \textbf{(3)}$$

A single electron travelling from one end of the wire to another will undergo a drop in potential energy, ΔE, given by:

$$\Delta E = eV \quad \textbf{(4)}$$

Due to this, the voltage used to compute the conductance is replaced by the relationship in Eq.4 to produce

$$G = ve^2 N/L\Delta E \quad \textbf{(5)}$$

To further resolve Eq.5, the number N of electrons travelling through a wire of length L, contributing to the conduction, as well as the difference in potential energy ΔE across the ends of the wire must be determined. First, the change in energy ΔE is given by the difference in Fermi energies between the terminals, which causes current to flow. Therefore, the Pauli exclusion principle can be invoked, which postulates that the Fermi energies can each occupy two electrons per state. As a result, the number of electrons N is twice the number of quantum states within ΔE.

The number of quantum states for this range of energies is derived using the quantum mechanics for a particle in a box of length L with electron velocities in a range Δv. Utilizing the de Broglie wavelength of an electron

$$\lambda_n = L/n \quad \textbf{(6)}$$

which takes on discrete values at the integers $n = 1, 2, 3, ...$ such that the velocity of electrons, with effective mass m,

$$v = h/\lambda m \quad \textbf{(7)}$$

has discrete states:

$$v_n = nh/Lm \quad \textbf{(8)}$$

For a velocity range Δv, Eq.8 is used to compute the number of quantum states,

$$n = Lm\Delta v/h \quad \textbf{(9)}$$

Recalling the relationship of this quantity with the number of electrons N, Eq. 9 is resolved to:

$$N = 2Lm\Delta v/h. \quad \textbf{(10)}$$

By considering the kinetic energy of an electron

$$E = mv^2/2 \quad \textbf{(11)}$$

It is deduced that for a range of velocities,

$$\Delta E = mv\Delta v. \quad \textbf{(12)}$$

Substituting Eq. 12 and the number of electrons contributing to conduction

$$N = 2L\Delta E/vh \quad \textbf{(13)}$$

with Eq. 5, the result is the quantum conductance

$$G = 2e^2h \quad \textbf{(14)}$$

which occurs at integer multiples.

Theory

The Quantum Hall Effect began the study of electron transport through conductors that revealed different integral factors and stepping behaviors. This effect is explored through mesoscopic systems.

Mesoscopic systems are systems intermediate in size, on a log scale, and are between microscopic and macroscopic. The primary feature of this type of system is its quantum wave nature of electrons. These systems work together with experimental techniques to reveal important electron transport features. Such as, its semiconducting materials are significant for showing quantum effects [4]. Even with those of differing material and size, or submergence in various fluids. Quantization is a universal property of any metallic contact.

An example of a mesoscopic system is quantized conductance. Quantized conductance is a measure of electricity flow which involves nanosized contact. The contact is on the scale of a few to a hundred atoms. When a constant voltage is applied to two gold wires, electricity flows as they vibrate coming in and out of touch with one another. This vibrational contact acts as a waveguide for electrons; an integer number of modes that are above cutoff will propagate in the waveguide [2]. The value for the number of modes allowed will change depending on the contact. The value will jump from one integer value to the next reminiscent of quantization.

Quantized conductance signifies that there will only be a finite number of occupied modes for the wave. As the size of the contact is changed, the number of modes that are allowed jumps discontinuously. And, for each mode, the nanoscale contact has the conductance derived from Eq.14, $G = 2e^2/h$, where, e, is electron charge and, h, is planck's constant.

In this experiment, two gold wires were vibrated for an applied voltage as the current decay is measured. The plot of this is known to show the discontinuous step pattern over time as quantized conductance is observed.

Apparatus: Scope

After examining various research studies as background, we decided to replicate the circuit depicted in Fig. 1 below [2].

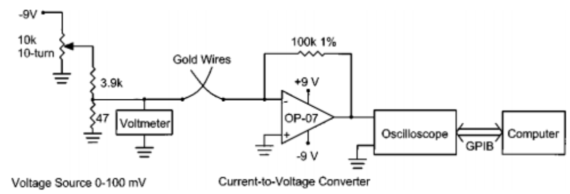


Fig. 1: Experimental setup that we modeled on our breadboard.

We considered the "right side" of this experiment to be the op-amp setup and everything to the right of it. The "left side", accordingly, refers to the gold wires and the resistor setup to the left of them. The right side of the apparatus begins with the oscilloscope connection into the computer, which is running the computational analysis tool called Waveforms, and the oscilloscope is grounded. Next, the op-amp, which refers to an Operational Amplifier, is connected in series with a 100k kOhm resistor connected in parallel around the op-amp. The op-amp has 8 pin connection points, which can be more clearly seen in Fig. 2 below. The circuit is connected to the second ("inverting input") pin, the third ("non inverting input") pin is grounded, the fourth ("negative voltage") pin is given a voltage supply of -9 V, the sixth ("output") pin is connected to the circuit, and lastly the seventh ("positive voltage") pin is given a voltage supply of 9 V.

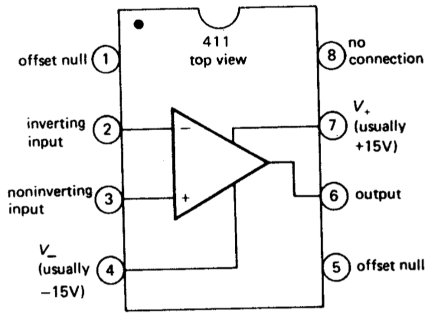


Fig. 2: Op-amp used on the right side of the circuit [3].

The op-amp provides a way for voltage gain over the circuit, resulting in the output voltage being much higher than the input voltage when fed through the op-amp. This voltage amplification is possible by using the op-amp with feedback, which we do by connecting the resistor across the op-amp and into the inverting input and the output pins. The purpose of using this voltage amplifying device is to emphasize the otherwise extremely small voltage differences that arise from this experiment. The weak electric signal from the brief moments of quantized conductance of the gold wires would be very hard to detect alone, so we amplify the response.

The left side of the circuit consists of the gold wires, set up so that they may vibrate between touching and not touching when the table is shaken, and the combination of resistors. The 47 Ohm and 3.9 kOhm resistors are connected in parallel, followed by a 10 kOhm 10-turn resistor. The 47 Ohm and 10 kOhm resistors are grounded. Lastly, a negative voltage supply is connected to the 10 kOhm 10-turn resistor at the very end of the circuit.

Apparatus: Application

Using the circuit kits provided to us by the Physics department, we built the previously described circuit using a breadboard, resistors, an op-amp, batteries, an oscilloscope, gold wires, and connection wires. The gold wires and oscilloscope can be seen in Figs. 3 and 4, respectively.

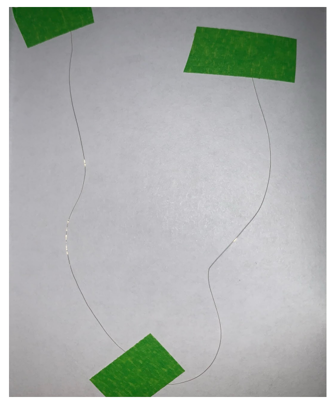


Fig 3: Gold wires used in the circuit.

Fig. 4: Analog Discovery 2 oscilloscope.

The right side of our circuit followed the previously described circuit almost exactly, and we used two batteries to supply the -9 V and 9 V to the op-amp pins, which can be seen in Fig. 1. Each battery was connected with one side to the op-amp, and the other side grounded. We used the Waveforms analysis software as the oscilloscope screen, where we could view the voltage relationship after causing the gold wires to shake, and change different viewing settings. The software also allowed us to apply a negative input supply voltage, which can be seen with the white wire labeled “-V” in Fig. 5.

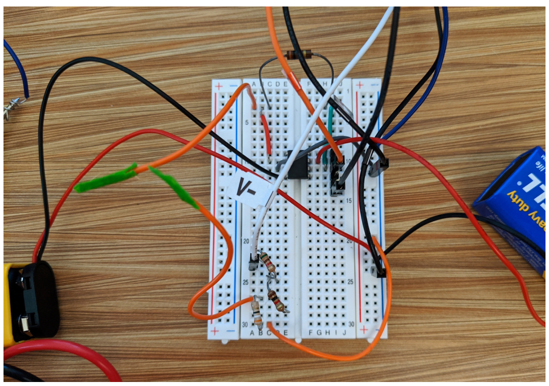


Fig. 5: The breadboard circuit made to model the experimental setup.

For the left side of the circuit, we altered the resistors a bit due to the supply that we had in order to achieve a similar small appliance of resistance. The resistors used are 5.1k Ohms, 1k Ohms, and 39 Ohms.

Procedure

Our first step was to test the workings of our circuit. We started with the right side of the circuit, and set up the op-amp system with the 100k Ohm resistor and oscilloscope connection into our computers. We tested this by then applying some voltage to the op amp and manually testing the resistor with a multimeter to estimate the response that we should get. We then added the gold wires to the circuit, and made sure that we had a voltage output of about 9 V on the oscilloscope when they were fully touching, and an output at 0 V when they were not touching. We then applied the rest of the left side of the circuit, including the resistors. Next, we took multiple runs for various different input voltages. For each run, we lightly hit the table that the breadboard circuit was on in order to shake the gold wires as Waveforms was running. We observed and saved the stepping relationship of quantized conductance between the gold wires as they went in and out of connection. The stepping relationship of conductance is observed as the step in voltage on the oscilloscope. We took five of these runs for five different input voltages: -2.25 V, -2.50 V, -2.75 V, -3.00 V, and -3.25 V. From there, we then randomly chose 3 of those runs from each input voltage, resulting in a total of 15 runs to analyze.

After collecting this data, we then analyzed it by creating histograms for each input voltage in order to observe the peaks due to the stepping relationship. We then divided out the factor of input voltage in order to get a master histogram from all 15 runs, which will be detailed further in the Analysis section. Lastly, we calculated what our expected step voltage locations would be based off of the gold wire conductance, and compared them to our observed experimental step locations.

Data

The data was taken on Waveforms Oscilloscope using the same circuit set up with different applied voltages to observe conductance responses. We collected numerical data for voltage and time and visually observed conductance steps. The theoretical step size voltage was calculated using:

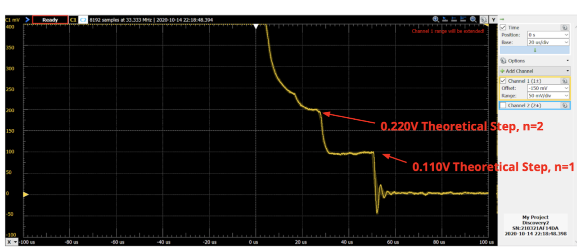
$$V_{into\ wire} * G * 100k\,\Omega \qquad \textbf{(15)}$$

The values for the first step size of each input voltage is shown in Fig. 6 below. To obtain higher values, you use integer multiples of the conductance n=1,2,3.

Waveforms Input Voltage (V)	Voltage Into Gold Wire (mV)	**Step size (V) for n = 1**	**Step size uncertainty**
-2.5	-15.8 ± 0.1	**0.122**	0.001
-2.25	-14.2 ± 0.1	**0.110**	0.001
-2.75	-17.4 ± 0.1	**0.135**	0.002
-3	-18.9 ± 0.1	**0.146**	0.002
-3.25	-20.5 ± 0.1	**0.159**	0.002

Fig 6: Five input voltages were applied to the system on Waveforms. For each system the voltage into the gold wire was measured with a multimeter at the 39 Ohm resistor.

The applied voltages and their corresponding oscilloscope are shown below in Fig. 7(a)-(e). Labelled are the theoretical step sizes where the measured step size occurred. The data from the voltages of the oscilloscope was made into a corresponding histogram below each oscilloscope trace.



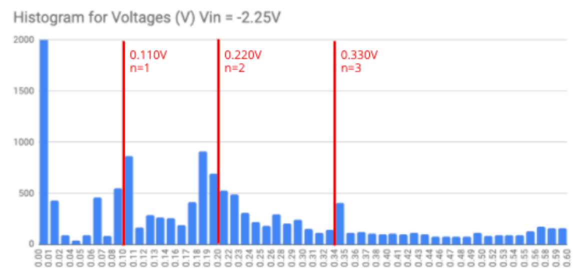
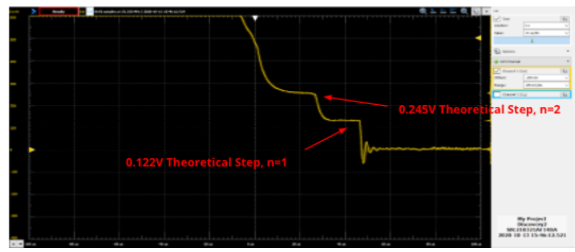
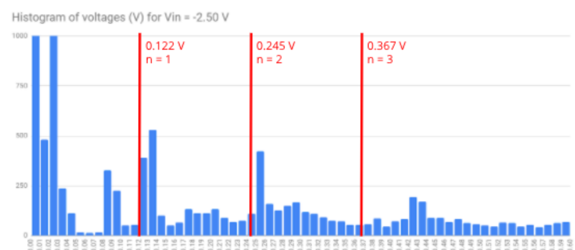


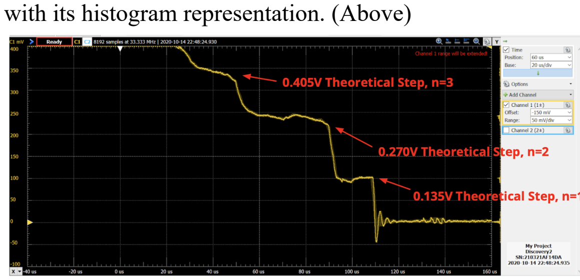
Fig. 7: Data from the oscilloscope trace and values in a histogram.

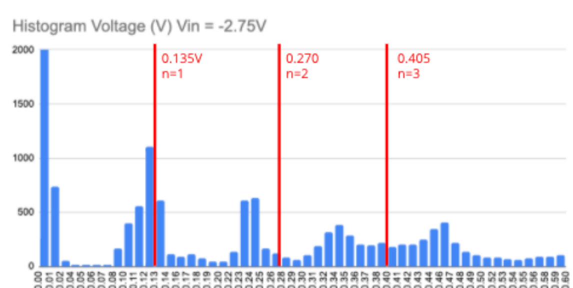
(a) -2.25V input on the Waveforms oscilloscope. The theoretical step-sizes are labelled for each step on the trace with its histogram representation. (Above)



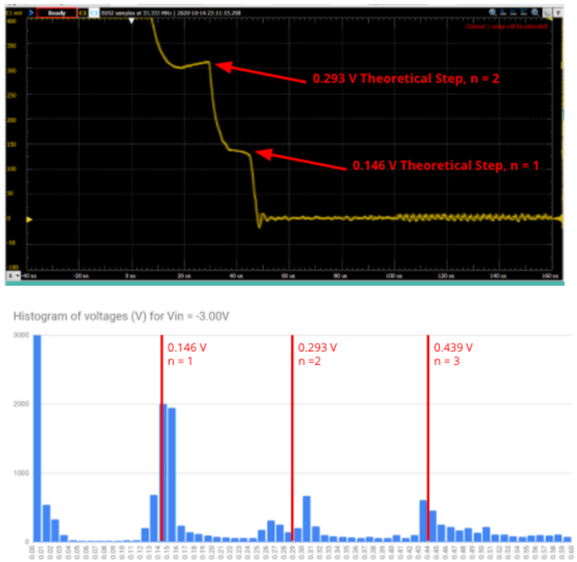


(b) -2.50V input on the Waveforms oscilloscope. The theoretical step-sizes are labelled for each step on the trace with its histogram representation. (Above)

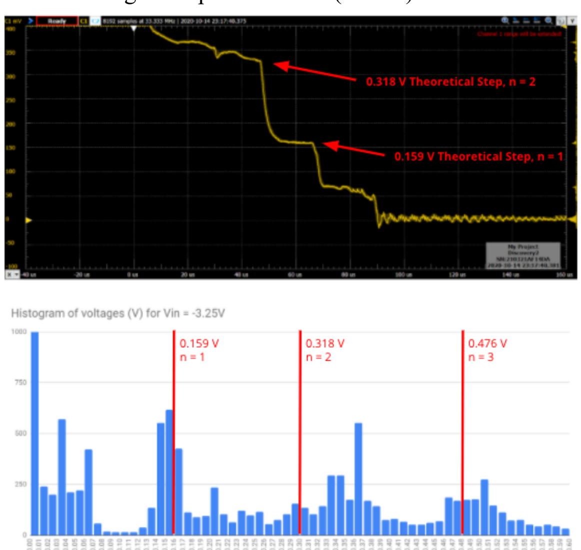




(c) -2.75V input on the Waveforms oscilloscope. The theoretical step-sizes are labelled for each step on the trace with its histogram representation. (Above)



(d) -3.00V input on the Waveforms oscilloscope. The theoretical step-sizes are labelled for each step on the trace with its histogram representation. (Above)



(e) -3.25V input on the Waveforms oscilloscope. The theoretical step-sizes are labelled for each step on the trace with its histogram representation. (Above)

Uncertainties

A few uncertainties arose from this experiment. Firstly, as with any electrical signal, there was experimental noise. This can be seen in Fig. 8, where the signal oscillates at zero on the oscilloscope trace.

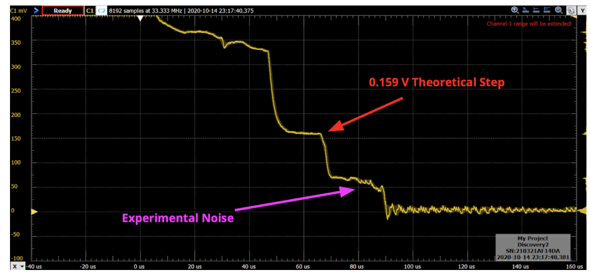


Fig. 8 : -3.25V input oscilloscope trace oscillating around 0V.

This resulted in a few bins around zero in the histograms to be filled, rather than right at zero. Some steps were also not as clear as others, as pointed out in Fig. 9 below, and these made the steps harder to identify because they didn't show up very well in the histograms.

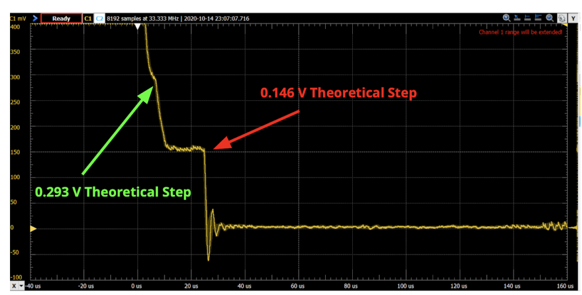


Fig. 9: -3.00 V input oscilloscope trace with not sharp steps above n=1.

Additionally, there was error on the multimeter measurements of voltage and resistance. There were also a few "bad runs", one of which is highlighted in Fig. 10 below. These runs created inaccuracies in the histograms, resulting in extra counts at various locations. These errors happened from various circuit malfunctions like loose wires.

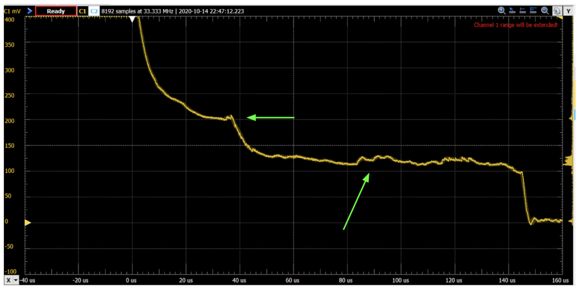


Fig. 10: -3.25 V input oscilloscope trace demonstrating a poor run.

Further error resulted from analysis calculations, including the calculated theoretical step size in Eq. 15. We needed to measure the voltage into the gold wire using a multimeter to find this step size. This value fluctuated $\pm 0.1V$ for each reading, accordingly this is the uncertainty for each voltage reading into the gold wire. The uncertainty on the resistor was found by performing a multimeter measurement of resistance. This measured value was compared to the actual value found using the color code on the resistor. The color code showed the resistor to be to 100KΩ, but our reading came out to be 99KΩ giving us an error of ± 1KΩ. Performing the calculation with these uncertainties we used error propagation for uncertainty on the theoretical step size shown in Fig. 11 below:

$$\frac{\Delta s}{s} = \sqrt{(\frac{\Delta V}{V})^2 + (\frac{\Delta R}{R})^2}$$

Input Voltage	Theoretical Step Size, n=1 (V)	Theoretical Step Size, n=2 (V)	Theoretical Step Size, n=3 (V)
-2.25V	0.110 ± 0.001	0.220 ± 0.001	0.330 ± 0.001
-2.5V	0.122 ± 0.001	0.244 ± 0.001	0.367 ± 0.001
-2.75V	0.135 ± 0.002	0.270 ± 0.002	0.405 ± 0.002
-3.00V	0.146 ± 0.002	0.293 ± 0.002	0.439 ± 0.002
-3.25V	0.159 ± 0.002	0.318 ± 0.002	0.476 ± 0.002

Fig. 11: Table of data and calculations for our theoretical step size with uncertainty. Theoretical step sizes are shown compared to our measured step sizes marked on the oscilloscopes and histograms in the data above.

Our measured step size is found from the peaks of our histograms for each applied voltage run. The peaks correspond to the step since there is a horizontal trace for a longer time interval for each step. The histogram bins are values between two numbers having a difference of 0.01. Therefore our uncertainty for each measured step size represented by the peak histogram bin is $\pm 0.01V$.

Input Voltage (V)	Measured Step Size, n=1 (V)	Measured Step Size, n=2 (V)	Measured Step Size, n=3 (V)
-2.25	0.105 ± 0.01	0.185 ± 0.01	0.345 ± 0.01
-2.50	0.135 ± 0.01	0.255 ± 0.01	0.425 ± 0.01
-2.75	0.125 ± 0.01	0.240 ± 0.01	0.345 ± 0.01
-3.00	0.150 ± 0.01	0.305 ± 0.01	0.435 ± 0.01
-3.25	0.155 ± 0.01	0.365 ± 0.01	0.505 ± 0.01

Fig. 12: Table of data and calculations for our measured step size with uncertainty for a bin width of 0.01V.

Analysis and Discussion

Our focus was to observe evidence of quantized conductance by experimentally comparing our measured step size for each integer value of conductance, $G = 2e^2/h$, to the theoretical value calculated using Eq. 15. Through our oscilloscope traces, we changed the applied voltage to observe steps as the wires vibrated together in Fig. 7 plots (a)-(e). A corresponding histogram of the voltage data was made to observe peaks for conductance modes. In Fig. 11 we recorded our values for the theoretical step size voltages we expected using our data and in Fig. 12 are the measured step sizes from the histograms. By averaging the theoretical step sizes for each integer multiple of conductance we are able to see our percent accuracy of our measurement:

$$\%error = \frac{|theoretical\ value - measured\ value|}{theoretical\ value}$$

For integer n=1, our theoretical step size average is $s_t = 0.134 \pm 0.002V$ and our measured step size is $s_m = 0.134 \pm 0.01V$. For integer n=2, our theoretical step size average is $s_t = 0.269 \pm 0.002V$ and our measured step size is $s_m = 0.270 \pm 0.01V$. For integer n=3, our theoretical step size average is $s_t = 0.403 \pm 0.002V$ and our measured step size is $s_m = 0.411 \pm 0.01V$. Using these values our percent error, we were able to measure the step size for n=1 with 0% error. For n=2 the step size was measured with 0.37% error and for n=3 the step size was measured with 1.95% error. With this, we see that we are able to measure the step size for smaller integer multiples of conductance more accurately as the percent error for n=3 is the largest.

The data taken for all input voltages was used to solve for conductance, $G = \frac{I}{V}$, where the I is the current through the gold wire as a function of time, $I(t) = \frac{V(t)}{R}$ and V is the voltage into the gold wire. Here we used the resistance of the resistor in the amplifier loop of the circuit, $R = 99k\,\Omega$, measured by the multimeter. We combine these relationships to find the conductance values for data across all input voltages as $G(t) = \frac{V(t)}{R} * \frac{1}{V_{into\ gold\ wire}}$. In Fig. 13, the conductance is compiled into a histogram. Using R in RStudio IDE, the histogram was made with bin width is $1.32 * 10^{-5}$ and was plotted with a fitted curve to the bin counts.

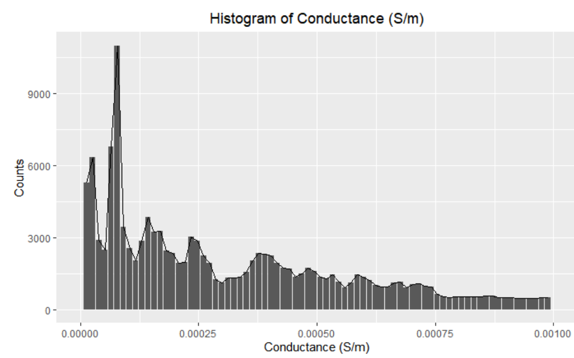


Fig. 13: Histogram of conductance data for all input voltages with a fitted curve to the bin counts.

The histogram data was then input to Python with the Spyder IDE to execute a spline fit to better visualize the data in Fig. 14. This fit shows the peaks from the histogram data above as red markers. The peaks from the histogram were then used to find the measured quantum conductance G_0.

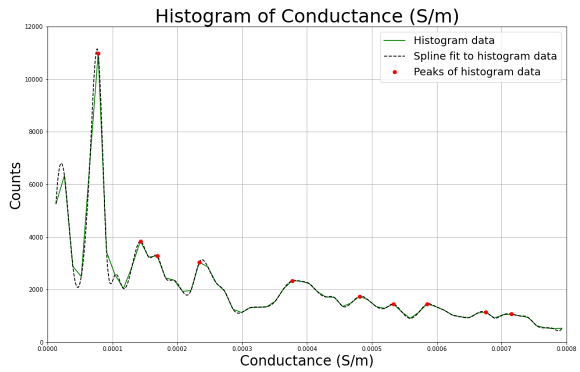


Fig. 14: Histogram of conductance to represent the peaks shown with red markers along the fit.

The histogram in Fig. 14 shows the first tallest peak conductance value to be G_0. Following this we see smaller peaks for multiples of G_0 that are slightly shifted due to residual resistance in series with the gold wires. We are able to compute the corrected conductance, G_c, using this value:

$$G_c = (G^{-1} - (R_{res} + R_{out}))^{-1}$$

G is the measured conductance, $R_{out} = 39\Omega$, the output resistance of the voltage source, and R_{res} is the residual resistance that we try different values for. We perform a minimum chi square estimation of R_{res} in Fig. 15 to get the corrected data. The program iterated through values of R_{res}. At each value, the corrected quantum conductance, G_c, were computed. For each G_c, the possible G_0 values were iterated through and the chi-square was computed with respect to G_c. Then, the minimum of all these chi square values is returned. Of all the Chi-square minimums, we select the smallest chi-square which corresponds to the optimal $R_{res} = 218.57\ Ohms$ and the optimal G_0.

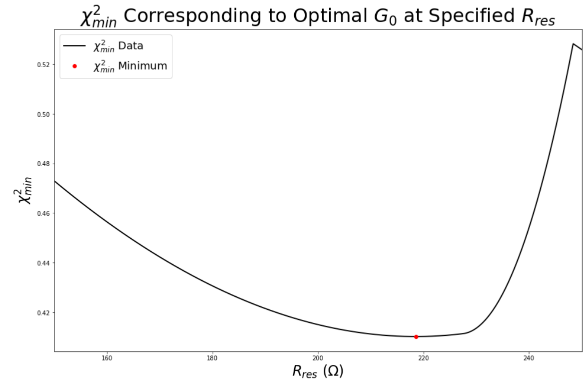


Fig. 15: Chi square minimum for various values of R_{res} corresponding to optimal conductance.

Using the R_{res} we found by minimizing the chi-square, we plot the chi square versus various possible values of G_0, which are shown above in Fig. 14 between 0.000075 and 0.000080 with N=1000 points. This graph is shown below in Fig. 16 to find where the quantum conductance has a chi square shown by the red marker.

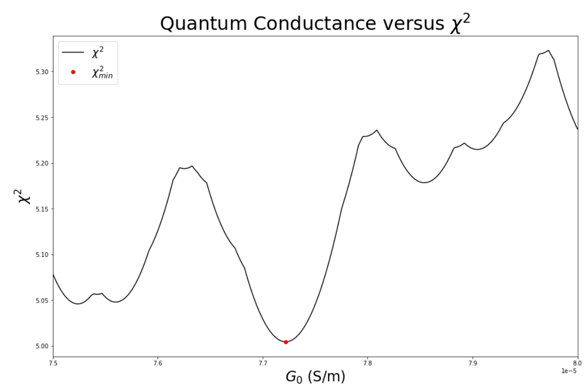


Fig. 16: Quantum conductance versus chi square.

The minimum corresponds to the measured quantum conductance, $G_0 = 7.72 * 10^{-5}$. Our theoretical value was $G = 2e^2/h = 7.75 * 10^{-5}$. By adding the bin width, $1.32 * 10^{-5}$, as error on our peaks in the histogram Fig. 14, we get a corresponding value of $G_0 = 7.64 * 10^{-5}$. The difference between this and the previous value $G_0 = 7.72 * 10^{-5}$ is $0.08 * 10^{-5}$, hence the error on our measured value making . $G_0 = 7.72 * 10^{-5} \pm 0.08 * 10^{-5}$. Thus, the theoretical value agrees with our results as it is within our measured value within uncertainty.

We can now plot the spline fit histogram in Fig. 14 with the corrected conductance, G_c for $R_{res} = 218.57\Omega$ in Fig. 17.

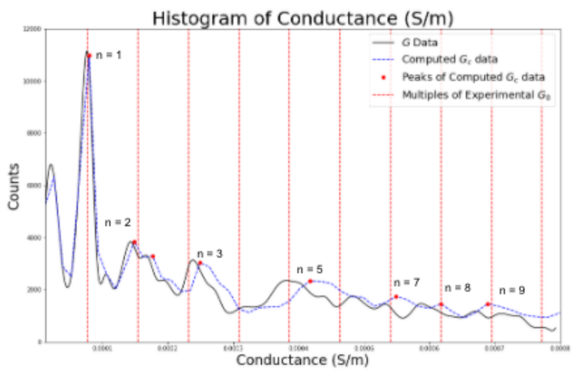


Fig. 17: Comparing the peaks of conductance for our corrected value with the integer multiples of G_0.

From Fig. 17, we can compare the peaks of G_c with the multiples of the measured quantum conductance, G_0. The peaks of our corrected conductance are shown with red markers and the multiples, n=1,2,3… , are represented by the vertical red dotted lines. We observe that the first three integer multiples match fairly well to the corrected quantum conductance.

We then compute residuals as the difference between the multiples of G_0 and its closest peak of G_c in Fig. 18. This signifies how close our model is to the data.

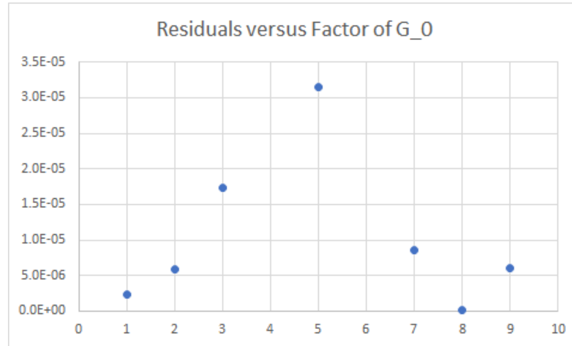


Fig 18: Plot of residuals for the difference between the multiples of G_0 and its closest peak of G_c.

For the integer multiple n=1, we are $+0.2 * 10^{-5}$ away from our theoretical value and for multiple n=2 we are $+0.6 * 10^{-5}$ away from our theoretical value. According to Fig. 18, the last integer multiple we can trust is n=3 as there is no value close to n=4 in Fig. 17. This makes inconsistent data beyond this point.

Conclusion

We conclude that quantized conductance has a stepping relationship for two gold wires that come in and out of contact with each other with an applied voltage. We were able to calculate the step sizes of the voltage with low percent error from making histograms of the oscilloscope data. Our measured step values were closer to the theoretical value for lower integer multiples of conductance. We solved for the conductance of all input voltages and compiled the data into a histogram. By fitting the histogram and marking its peaks, we were able to find the possible values for measured conductance. Using the measured conductance and the output resistance, we iterated through values of residual resistance to solve for the corrected value of conductance. With a minimum chi square fit, we found the residual resistance that corresponds to the optimal value of our measured quantum conductance. Knowing our possible values of our measured conductance from the previously made histogram, we plotted the chi square versus the possible values to find the minimum. We find the difference between the minimum value of our measured conductance and subtract it with the value we get from adding error on our peaks in the histogram. This difference is the uncertainty of the minimum chi square measured value for quantum conductance. We are able to compare this value, $G_0 = 7.72 * 10^{-5} \pm 0.08 * 10^{-5}$, to our theoretical value, $G = 2e^2/h = 7.75 * 10^{-5}$. We conclude that our measured value agrees to the theoretical value as it is within the uncertainty of our measured value. Plotting a spline fit histogram with the corrected conductance and comparing its peaks to the conductance multiples of the measured conductance, we made a residual plot to show how close our fit is to the data. The residual plot showed our data for the first three integer multiples of conductance are accurate as shown earlier by the percent error.

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